

An alternative strategy for factorising quadratics

The example below demonstrates a strategy for factorising quadratics that's an alternative to the strategy given in Subsection 4.2 of MST124 Unit 2. It works for any quadratic that can be factorised using integers, but it's particularly useful for quadratics in which the coefficient of x^2 is not 1. The explanation of why the strategy works is complicated and not included here.

Example 1 *Factorising a quadratic – alternative method*

Factorise the quadratic $2x^2 - x - 6$.

Solution

 Identify the values of a , b and c (thinking of the quadratic as $ax^2 + bx + c$). 

Here $a = 2$, $b = -1$ and $c = -6$.

 Find a pair of numbers whose product is ac and whose sum is b . 

We have $ac = 2 \times (-6) = -12$ and $b = -1$.

The factor pairs of -12 are

$-1, 12, \quad 1, -12, \quad -2, 6, \quad 2, -6, \quad -3, 4, \quad 3, -4$.

Of these, a pair with sum -1 is $3, -4$.

 Rewrite the quadratic expression, splitting the term in x using the pair of numbers found. 

Since $-1 = 3 - 4$,

$$2x^2 - x - 6 = 2x^2 + 3x - 4x - 6$$

 Group the four terms as two pairs and take out a common factor within each pair. 

$$\begin{aligned} &= \underline{2x^2 + 3x} \quad \underline{-4x - 6} \\ &= x(2x + 3) - 2(2x + 3) \end{aligned}$$

 Take out the common factor of the resulting terms (the linear expression in brackets). 

$$= (x - 2)(2x + 3).$$

The method works no matter which order you choose when you split the middle term. For example, since $-1 = -4 + 3$,

$$\begin{aligned}2x^2 - x - 6 &= 2x^2 - 4x + 3x - 6 \\&= 2x(x - 2) + 3(x - 2) \\&= (2x + 3)(x - 2).\end{aligned}$$